ENERGY-MOMENTUM OF THE GRAVITATIONAL FIELD IN THE TELEPARALLEL GEOMETRY

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Abstract

The Hamiltonian formulation of the teleparallel equivalent of general relativity (TEGR) without gauge fixing has recently been established in terms of the Hamiltonian constraint and a set of six primary constraints. Altogether, they constitute a set of first class constraints. In view of the constraint structure we establish definitions for the energy, momentum and angular momentum of the gravitational field. In agreement with previous investigations, the gravitational energy-momentum density follows from a total divergence that arises in the constraints. This definition is applied successfully to the calculation of the irreducible mass of the Kerr black hole. The definition of the angular momentum of the gravitational field follows from the integral form of primary constraints that satisfy the angular momentum algebra.

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The dynamics of the gravitational field can be described in the context of the teleparallel geometry, where the basic geometrical entity is the tetrad field $e^a\,_\mu$, (a and μ are SO(3,1) and space-time indices, respectively). Teleparallel theories of gravity are defined on the Weitzenböck space-time[1], endowed with the affine connection

$$\Gamma^\lambda_{\mu\nu} = e^{a\lambda}\partial_\mu e_{a\nu}\ . \tag{1}$$

The curvature tensor constructed out of (1) vanishes identically. This connection defines a space with teleparallelism, or absolute parallelism[2]. This geometrical framework was considered by Einstein[3] in his attempt at unifying gravity and electromagnetism.

In the teleparallel geometry it is possible to establish an alternative description of Einstein's equations. Such description is given by the teleparallel equivalent of general relativity (TEGR)[4, 5, 6, 7, 8, 9, 10, 11]. Gravity theories in this geometrical framework are constructed out of the torsion tensor $T^a\,_{\mu\nu} = \partial_\mu e^a\,_\nu - \partial_\nu e^a\,_\mu$, which is related to the antisymmetric part of (1). Interesting features of the TEGR take place in the Hamiltonian framework.

The Hamiltonian formulation of the TEGR has been obtained in Ref. [10]. In the latter, however, the time gauge was fixed from the outset. As a consequence of this gauge fixing the teleparallel geometry is restricted to the three-dimensional spacelike hypersurface.

In the framework of the TEGR it is possible to make definite statements about the energy and momentum of the gravitational field. A simple expression for the gravitational energy arises in the Hamiltonian formulation of the TEGR[10] in the framework of Schwinger's time gauge condition[12]. The energy density is given by a scalar density in the form of a total divergence that appears in the Hamiltonian constraint of the theory[13]. By applying this definition to several configurations of the gravitational field encouraging and satisfactory results have been obtained. The investigations carried out so far confirm the consistency and relevance of this energy expression.

The Hamiltonian formulation of the TEGR without gauge fixing has recently been established[14]. Its canonical structure is different from that obtained in Ref. [10], since it is not given in the standard ADM form[15]. In fact it has not been necessary to establish the usual 3+1 decomposition for the metric and tetrad fields. In this framework we again arrive at an expression for the gravitational energy, in strict similarity with the procedure adopted

in Ref. [13], namely, by interpreting the Hamiltonian constraint equation as an energy equation for the gravitational field. Likewise, the gravitational momentum can be defined. The gravitational energy-momentum arises as a SO(3,1) vector. The constraint algebra of the theory suggest that certain momenta components are related to the gravitational angular momentum. It turns out to be possible to define, in this context, the angular momentum of the gravitational field.

In this paper we investigate the definitions of gravitational energy and angular momentum that arises in Ref. [14] in the framework of the Kerr metric. We recall that the whole formulation developed in Ref. [14] is carried out without enforcing the time gauge condition. It turns out, however, that consistent values for the gravitational energy are achieved by requiring the tetrad field to satisfy the time gauge condition. This amounts to *a posteriori* restriction on the tetrads.

Notation: spacetime indices $\mu, \nu, ...$ and SO(3,1) indices $a, b, ...$ run from 0 to 3. Time and space indices are indicated according to $\mu = 0, i, \quad a = (0), (i)$. The flat, Minkowski spacetime metric is fixed by $\eta_{ab} = e_{a\mu} e_{b\nu} g^{\mu\nu} = (-+++)$.

The Lagrangian density of the TEGR in empty space-time is given by[10, 14]

$$L(e) \;=\; -k\, e \left(\frac{1}{4} T^{abc} T_{abc} + \frac{1}{2} T^{abc} T_{bac} - T^a T_a \right) , \tag{2}$$

where $k = \frac{1}{16\pi G}$, G is Newton's constant, $e = det(e^a{}_\mu)$ and $T_{abc} = e_b{}^\mu e_c{}^\nu T_{a\mu\nu}$. Tetrads transform space-time into SO(3,1) indices and vice-versa. The trace of the torsion tensor is given by $T_b = T^a{}_{ab}$.

In the Hamiltonian formulation developed in Ref. [14] it has not been made use of any kind of projection of metric variables to the three-dimensional spacelike hypersurface. The Hamiltonian was obtained by just rewriting the Lagrangian density in the form $L = p\dot{q} - H$. Since there is no time derivative of e_{a0} in (2), the corresponding momentum canonically conjugated Π^{a0} vanishes identically. Dispensing with surface terms the total Hamiltonian density reads[14]

$$H(e_{ai}, \Pi^{ai}) = H_0 + \alpha_{ik} \Gamma^{ik} + \beta_k \Gamma^k \ . \tag{3}$$

The Hamiltonian constraint H_0 and the primary constraits Γ^{ik} and Γ^k are given by, respectively,

$$H_0 \;=\; -e_{a0}\partial_k\Pi^{ak} - \frac{1}{4g^{00}}ke\Big(g_{ik}g_{jl}P^{ij}P^{kl} - \frac{1}{2}P^2\Big)$$
$$+ke\Big(\frac{1}{4}g^{im}g^{nj}T^a{}_{mn}T_{aij} + \frac{1}{2}g^{nj}T^i{}_{mn}T^m{}_{ij} - g^{ik}T^j{}_{ji}T^n{}_{nk}\Big)\;, \tag{4}$$

$$\Gamma^{ik} \;=\; -\Gamma^{ki} \;=\; \Pi^{[ik]} \;-\; k\,e\Big(-g^{im}g^{kj}T^0{}_{mj} + (g^{im}g^{0k} - g^{km}g^{0i})T^j{}_{mj}\Big)\;, \tag{5}$$

$$\Gamma^k \;=\; \Pi^{0k} \;+\; 2k\,e\,(g^{kj}g^{0i}T^0{}_{ij} - g^{0k}g^{0i}T^j{}_{ij} + g^{00}g^{ik}T^j{}_{ij})\;. \tag{6}$$

(..) and [..] denote symmetrization and anti-symmetrization, respectively. In (4) we have the following definitions

$$P^{ik} \;=\; \frac{1}{ke}\Pi^{(ik)} - \Delta^{ik}\;, \tag{7}$$

$$\Delta^{ik} \;=\; -g^{0m}(g^{kj}T^i{}_{mj} + g^{ij}T^k{}_{mj} - 2g^{ik}T^j{}_{mj}) - (g^{km}g^{0i} + g^{im}g^{0k})T^j{}_{mj}\;, \tag{8}$$

and $P = g_{ik}P^{ik}$.

It has been shown[14] that the Lagrange multipliers α_{ik} and β_k are determined from the evolution equations, $\alpha_{ij} = \frac{1}{2}(T_{i0j} - T_{j0i})$, $\beta_j = T_{00j}$, and that although e_{a0} is present within the structure of H_0, Γ^{ik} and Γ^k, it actually plays the role of a Lagrange multiplier (see equation (10) below).

The vanishing of the momentum canonically conjugated to e_{a0}, Π^{a0}, induces the secondary constraints

$$C^a(x) \equiv \frac{\delta H}{\delta e_{a0}(x)} = 0\;, \tag{9}$$

which satisfy[14]

$$e_{a0}C^a = H_0\;. \tag{10}$$

It is possible to show that C^a may be written in a simplified form as

$$C^a = e^{a0} H_0 + e^{ai} F_i \ , \tag{11}$$

where

$$F_i = H_i + \Gamma^m T_{0mi} + \Gamma^{lm} T_{lmi} + \frac{1}{2g^{00}} (g_{ik} g_{jl} P^{kl} - \frac{1}{2} g_{ij} P) \Gamma^j \ . \tag{12}$$

The vector constraint H_i is given by

$$H_i = -e_{bi} \partial_k \Pi^{bk} - \Pi^{bk} T_{bki} \ . \tag{13}$$

Therefore if H_0 vanishes, $e_{a0} C^a$ also vanishes. Since $\{e_{a0}\}$ are arbitrary, it follows that C^a vanishes as well. If in addition we have $\Gamma^{ij} = \Gamma^j = 0$, then we also have $H_i = 0$. Consequently the vanishing of H_i at any instant of time follows from the vanishing of H_0, Γ^{ij} and Γ^j at the same instant. Furthermore H_i is *derived* from H_0 in the subspace of the phase space determined by $\Gamma^{ij} = \Gamma^j = 0$,

$$e_{ai} \frac{\delta}{\delta e_{a0}} H_0 = H_i \ . \tag{14}$$

The Poisson bracket between two quantites F and G is defined by

$$\{F, G\} = \int d^3x \biggl(\frac{\delta F}{\delta e_{ai}(x)} \frac{\delta G}{\delta \Pi^{ai}(x)} - \frac{\delta F}{\delta \Pi^{ai}(x)} \frac{\delta G}{\delta e_{ai}(x)} \biggr) \ ,$$

by means of which we can write down the evolution equations and evaluate the constraint algebra.

The calculations of the Poisson brackets between the constraints (4), (5) and (6) are exceedingly complicated. We will just present the results. The constraint algebra is given by

$$\{H_0(x), H_0(y)\} = 0 \ , \tag{15}$$

$$\{\Gamma^i(x), \Gamma^j(y)\} = 0 \ , \tag{16}$$

$$\{\Gamma^{ij}(x), \Gamma^{kl}(y)\} = \frac{1}{2} \biggl(g^{il} \Gamma^{jk} + g^{jk} \Gamma^{il} - g^{ik} \Gamma^{jl} - g^{jl} \Gamma^{ik} \biggr) \delta(x - y) \ , \tag{17}$$

$$\{\Gamma^{ij}(x), \Gamma^k(y)\} = (g^{0j}\Gamma^{ki} - g^{0i}\Gamma^{kj})\delta(x-y) \ , \tag{18}$$

$$\{H_0(x), \Gamma^{ij}(y)\} = \left[\frac{1}{2g^{00}}P^{kl}\left(\frac{1}{2}g_{kl}g_{mn} - g_{km}g_{nl}\right)\left(g^{mi}\Gamma^{nj} - g^{mj}\Gamma^{ni}\right) + \right.$$

$$\left. + \frac{1}{2}\left(\Gamma^{nj}e^{ai} - \Gamma^{ni}e^{aj}\right)\partial_n e_{a0}\right]\delta(x-y) \ , \tag{19}$$

$$\{H_0(x), \Gamma^i(y)\} = \left[g^{0i}H_0 + \frac{1}{g^{00}}P^{kl}\left(\frac{1}{2}g_{kl}g_{jm} - g_{kj}g_{ml}\right)g^{0j}\Gamma^{mi}\right.$$

$$+ \left(\Gamma^{ni}e^{a0} + \Gamma^n e^{ai}\right)\partial_n e_{a0} + \frac{1}{2}\Gamma^{mn}T^i{}_{nm}$$

$$\left. + 2\partial_n\Gamma^{ni} + g^{in}\left(H_n - \Gamma^j T_{0nj} - \Gamma^{mj}T_{mnj}\right)\right]\delta(x-y)$$

$$+ \Gamma^{ni}(x)\frac{\partial}{\partial x^n}\delta(x-y) \ . \tag{20}$$

We note the presence of H_i on the right hand side of (20). However it poses no problem for the consistency of the constraints provided H_0, Γ^{ik} and Γ^k are taken to vanish at the intial time $t = t_0$. Let $\phi(x^i, t)$ represent any of the latter constraints. At the initial time we have $\phi(x^i, t_0) = 0$. At $t_0 + \delta t$ we find $\phi(x^i, t_0 + \delta t) = \phi(x^i, t_0) + \dot{\phi}(x^i, t_0)\delta t$ such that $\dot{\phi}(x^i, t_0) = \{\phi(x^i, t_0), \mathbf{H}\}$, where $\mathbf{H}$ is the total Hamiltonian. Since the vanishing of H_i at an instant of time is a consequence of the vanishing of H_0, Γ^{ik} and Γ^k at the same time, the consistency of the constraints is guaranteed at any $t > t_0$.

One of the main motivations for studying the TEGR is that the constraint equations of the theory can be interpreted as energy-momentum equations. The definition of the gravitational energy given in Ref. [13] was motivated by interpreting the Hamiltonian constraint equation $C = 0$ of Ref. [10] as an equation of the type $C = H - E = 0$. A stringent application of such definition has been made in the context of rotating black holes[16]. In similarity with Ref. [13], in the present framework again we interpret the $a = 0$ component of the constraint equations $C^a = (0)$ as an energy equation for the gravitational field.

The total divergence that appears in C^a is given by $-\partial_i \Pi^{ai}$. After implementing the primary constraints (5) and (6) the momenta Π^{ak} reads

$$\Pi^{ak} \;=\; k\,e\Big\{g^{00}(-g^{kj}T^a{}_{0j} - e^{aj}T^k{}_{0j} + 2e^{ak}T^j{}_{0j})$$

$$+g^{0k}(g^{0j}T^a{}_{0j}+e^{aj}T^0{}_{0j}) + e^{a0}(g^{0j}T^k{}_{0j}+g^{kj}T^0{}_{0j}) - 2(e^{a0}g^{0k}T^j{}_{0j}+e^{ak}g^{0j}T^0{}_{0j})$$

$$-g^{0i}g^{kj}T^a{}_{ij} + e^{ai}(g^{0j}T^k{}_{ij} - g^{kj}T^0{}_{ij}) - 2(g^{0i}e^{ak} - g^{ik}e^{a0})T^j{}_{ji}\Big\}\,. \qquad (21)$$

We identify $-\partial_i \Pi^{ai}$, which is the first term in the expression of C^a, as the *energy-momentum density* of the gravitational field. The total energy-momentum is defined by

$$P^a = -\int_V d^3x \partial_i \Pi^{ai}\;. \qquad (22)$$

where V is an arbitrary space volume. It is invariant under coordinate transformations on the three-dimensional spacelike hypersurface, and transforms as a vector under the global SO(3,1) group. The definition above generalizes the expression previously obtained in Ref. [13] to tetrad fields that are not restricted by the time gauge condition.

In analogy with the previous analysis of Ref. [13], and following Møller[4], in the case of asymptotically flat space-times we may adopt the boundary conditions

$$e_{a\mu} \simeq \eta_{a\mu} + \frac{1}{2}h_{a\mu}(\frac{1}{r})\;, \qquad (23)$$

in the limit $r \to \infty$. In the expression above $\eta_{a\mu}$ is the Minkowski metric tensor and $h_{a\mu} = h_{\mu a}$ is the first term of the asymptotic expansion of the metric tensor $g_{\mu\nu}$. Since $h_{a\mu}$ is asymptotically a symmetric tensor, these boundary conditions impose 6 conditions on the tetrads. These conditions are not fixed in the body of the theory because the SO(3,1) is a global (rather than local) symmetry group (Møller called it *supplementary conditions*).

Similar conditions for triads restricted to the three-dimensional spacelike hypersurface were essential in order to arrive at the ADM energy[15]. In

the present approach we also obtain the ADM energy. Asymptotically flat spacetimes are defined by (23) together with $\partial_\mu g_{\lambda\nu} = O(\frac{1}{r^2})$, or $\partial_\mu e_{a\nu} = O(\frac{1}{r^2})$. Thus considering the $a = (0)$ component in (22) and integrating over the whole three-dimensional spacelike hypersurface we find that all terms of the type $T^a{}_{0j}$ cancel out, and eventually only the last term in (21) contributes to the integration. Hence we obtain

$$E \equiv -\int_{V\to\infty} d^3x \partial_k \Pi^{(0)k} = -2k \int_{V\to\infty} d^3x \partial_k (e g^{ik} e^{(0)0} T^j{}_{ji})$$
$$= \frac{1}{16\pi G} \int_{S\to\infty} dS_k (\partial_i h_{ik} - \partial_k h_{ii}) = E_{ADM} \ . \qquad (24)$$

The energy expression above can be applied to *finite* volumes of space. Therefore it can be used to obtain the irreducible mass of rotating black holes. It is the mass of the black hole at the final stage of Penrose's process of energy extraction, considering that the maximum possible energy is extracted. It is also the mass contained within the outer horizon of the black hole. Every expression for local or quasi-local gravitational energy must necessarily yield the value of M_{irr} in the calculation of the energy contained within the outer event horizon, since we know beforehand its value as a function of the initial angular momentum of the black hole[17]. The evaluation of M_{irr} is a crucial test for any expression for the gravitational energy (M_{irr} has been obtained by means of different energy expressions by Bergqvist[18]).

In terms of Boyer-Lindquist[19] coordinates the Kerr metric tensor is given by

$$ds^2 = -\frac{\psi^2}{\rho^2} dt^2 - \frac{2\chi sin^2\theta}{\rho^2} d\phi\, dt + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 + \frac{\Sigma^2 sin^2\theta}{\rho^2} d\phi^2 \ , \qquad (25)$$

where $\rho^2 = r^2 + a^2 cos^2\theta$, $\Delta = r^2 + a^2 - 2mr$ and

$$\Sigma^2 = (r^2 + a^2)^2 - \Delta a^2\, sin^2\theta \ ,$$

$$\psi^2 = \Delta - a^2\, sin^2\theta \ ,$$

$$\chi = 2amr \ .$$

In order to obtain M_{irr} we calculate the $a=(0)$ component of (22) by fixing V to be the volume within the $r_+ = constant$ surface, where r_+ is the outer horizon of the Kerr black hole,

$$r_+ = m + \sqrt{m^2 - a^2} \; .$$

Of course there is an infinity of tetrads that yield (25), but only one that leads to a viable expression for M_{irr}, and that gives the correct description of the gravitational energy. Here we will consider two sets of tetrads. The first one is taken to satisfy Møller's weak field approximation (23), and will be denoted by $e^M_{a\mu}$. It reads

$$e^M_{a\mu} = \begin{pmatrix} -\frac{\psi}{\rho}\sqrt{1+M^2y^2} & 0 & 0 & -\frac{\chi N y}{\psi\rho} sin^2\theta \\ \frac{\chi y}{\Sigma\rho} sin\theta\, sin\phi & \frac{\rho}{\sqrt{\Delta}} sin\theta\, cos\phi & \rho\, cos\theta\, cos\phi & -\frac{\Sigma}{\rho}\sqrt{1+M^2N^2y^2}\sin\theta\, sin\phi \\ -\frac{\chi y}{\Sigma\rho} sin\theta\, cos\phi & \frac{\rho}{\sqrt{\Delta}} sin\theta\, sin\phi & \rho\, cos\theta\, sin\phi & \frac{\Sigma}{\rho}\sqrt{1+M^2N^2y^2} sin\theta\, cos\phi \\ 0 & \frac{\rho}{\sqrt{\Delta}} cos\theta & -\rho\, sin\theta & 0 \end{pmatrix} , \tag{26}$$

where

$$y^2 = \frac{2N\sqrt{1+M^2} - (1+N^2)}{4M^2N^2 - (1-N^2)^2} \; ,$$

$$M = \frac{\chi}{\Sigma\psi} sin\theta \; ,$$

$$N = \frac{\psi r}{\Sigma} \; .$$

The second one satisfies Schwinger's time gauge condition,

$$e_{(k)}{}^0 = e^{(0)}{}_j = 0 \; , \tag{27a}$$

together with the weak field approximation

$$e_{(i)j} \simeq \eta_{ij} + \frac{1}{2} h_{ij} \; , \tag{27b}$$

$$h_{ij} = h_{ji} \; . \tag{27c}$$

It is given by

$$e^S_{a\mu} = \begin{pmatrix} -\frac{1}{\rho}\sqrt{\psi^2 + \frac{\chi^2}{\Sigma^2} sin^2\theta} & 0 & 0 & 0 \\ \frac{\chi}{\Sigma\rho} sin\theta\, sin\phi & \frac{\rho}{\sqrt{\Delta}} sin\theta\, cos\phi & \rho\, cos\theta\, cos\phi & -\frac{\Sigma}{\rho} sin\theta\, sin\phi \\ -\frac{\chi}{\Sigma\rho} sin\theta\, cos\phi & \frac{\rho}{\sqrt{\Delta}} sin\theta\, sin\phi & \rho\, cos\theta\, sin\phi & \frac{\Sigma}{\rho} sin\theta\, cos\phi \\ 0 & \frac{\rho}{\sqrt{\Delta}} cos\theta & -\rho\, sin\theta & 0 \end{pmatrix} . \tag{28}$$

Considering (21) and (22) we calculate the gravitational energy contained within a surface enclosing the black hole, determined by a constant radius r_0, and then take the limit $r_0 \to r_+$, both for $e^M_{a\mu}$ and for $e^S_{a\mu}$. The final expression arises as a function of the angular momentum per unit mass a.

By using (28) we find that the resulting energy expression, $E[e^S_{a\mu}]$, is precisely the same one obtained in Ref. [16], and therefore it agrees remarkably well with the expression of $2M_{irr}$ as a function of a. It reads

$$E[e^S_{a\mu}] = m\left[\frac{\sqrt{2p}}{4} + \frac{6p-k^2}{4k} ln\left(\frac{\sqrt{2p}+k}{p}\right)\right] , \tag{29}$$

where

$$p = 1 + \sqrt{1-k^2} \quad , \quad a = km, 0 \leq k \leq 1 .$$

The energy expression $E[e^M_{a\mu}]$ that is obtained by considering (26) deviates from M_{irr}. For values of $a \simeq 0.8m$ we find that $E[e^M_{a\mu}] > 2m$. However this value must be always less than or equal to $2m$. The expression is given by

$$E[e^M_{a\mu}] = \frac{m}{4}\int_0^\pi d\theta\, sin\theta \left[\sqrt{p^2 + k^2 cos\theta} + \frac{py}{\sqrt{p^2+k^2cos\theta}} \right.$$
$$\left. + \frac{2p^3 y}{(p^2+k^2cos\theta)^{\frac{3}{2}}} - \frac{y(p-1)\sqrt{p^2+k^2cos\theta}}{2}\right] . \tag{30}$$

Details will be presented elsewhere[20].

We will ascribe generality to the result above and assume that tetrads satisfying the time gauge condition (27a), together with (27b,c), yield the correct description of the gravitational energy.

The Poisson bracket (17) strongly suggestes that Γ^{ik} is related to the angular momentum of the gravitational field. In similarity with the definition

of the gravitational energy-momentum, we assume that the gravitational angular momentum M^{ik} is obtained from the integral form of the constraint equation $\Gamma^{ik} = 0$. Therefore we define

$$M^{ik} = \int d^3x\, \Pi^{[ik]}$$

$$= \int d^3x\, k\, e \Big[-g^{im} g^{kj} T^0{}_{mj} + (g^{im} g^{0k} - g^{km} g^{0i}) T^j{}_{mj} \Big] \,. \qquad (31)$$

Considering the Kerr space-time, the evaluation of M^{ik} out of (28) involves the evaluation of very intricate integrals. This issue is currently under investigation[20]. Here we just mention that the application of (31) to the simple case of the metric associated to a thin, slowly rotating mass shell as described by Cohen[21] yields the Newtonian expression for the angular momentum of the source. This result is obtained by integrating (31) over the *whole* three-dimensional space, and making use of the time gauge condition (27). It must be noted that such metric tensor corresponds to the asymptotic form of Kerr's metric tensor in the limit of small angular momentum.

Although the time gauge condition is clearly important in analysis of the Hamiltonian formulation of tetrad type theories of gravity, its relevance in the context of the present investigation is not fully understood. The analysis of Refs. [10, 13] was developed under the assumption of the time gauge condition, and as a consequence the teleparallel geometry was restricted to the three-dimensional spacelike hypersurface. By not assuming any *a priori* restriction on the tetrads the teleparallel geometry is extended to the four-dimensional space-time, and yet the time gauge condition continues to play a special role in the description of the energy of the gravitational field. The question regarding the relevance of the time gauge condition in the present analysis must be further investigated.

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